Research Ideas Escher

# Our strategy: allow for more general learning

The idea is to have a method that will be superior from the ones from the mathematicians in 2 aspects:

* Functional form - > It will allow for a more general family of functions (i.e. it will learn the best function from a given family)
* Coefficients - > It will allow for a more general parameters of the functions.

Both premises above will allow us to fit Escher’s lithography in a more exact way.  
  
The above is the contribution from our paper. For now, (as far as our current knowledge goes) we won’t be able to detach completely (or be superior) than the conformal mapping proposed by the mathematicians… we will be able to learn a more general conformal mapping and ideally… more tailored to the imperfections of the painting.

So I will start from the conformal mapping, in the sense that we still want a function that preserve angles, and I will propose some learning strategies, to give as much freedom to our algo as possible. While still being able to learn.

The Mathematician’s Conformal Mapping

The mathematicians’ conformal mapping is:

(1)

Where Z is the complex plane and ‘c’ is a constant in the imaginary domain

**How can we re-write the above in the most general possible way so that we expand the possibilities of the ML algo?**

# Option 1: S. Caenazzo’s Matrix Exponential Expansion.

<https://en.wikipedia.org/wiki/Matrix_exponential>

This is the most general of the options that I can think of:

(2)

Here ‘k’ is the order of the expantion and ‘c’ is the complex coefficient to learn.

**STRATEGY**:

* Start with a given K (i.e. assume a ‘generous’ order of expansion to start with) and learn ‘c’.  
  Then we can calibrate ‘K’ easily by analyzing the error term, or the extra terms that don’t bring any value. The ‘K’ term is usually a non-issue.
* We can code it in python and see if it works (i.e. if it is really a good approximation for Z^c)

**PROS:** we only have to learn 1 parameter, and the functional form assumed is very minimal (we still start by assuming a conformal map).

# Option 2: Hybrid approach, in case the above doesn’t work

Here we just expand a little bit more, to give one extra operation more explicitly. We include the Log(z) to be more specific to the ML algo.

(3)

**Again given an initial ‘k’ learn ‘c’**. Then same strategy as before, we can disregard the K terms that go to zero.

# Option 3: The Reddit Espansion.

This is an alternative representation of (1). It give us a representation with sine and cosines which can be justified as a pseudo-rotation matrix.

Use polar form of complex number

(4)

**STRATEGY**:

* learn c
* r? and theta? 🡪 we will need to somehow plug in ‘r’ and theta = arctan(y/x). Seems very laborious or I am not sure what to do with these terms.

# Option 4: S. Caenazzo’s 3-operations approach.

Another potential way to use the matrix exponential would be to use it as one of the three steps in the space warping:

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